ISSN 0005-1179 (print), ISSN 1608-3032 (online), Automation and Remote Control, 2023, Vol. 84, No. 9, pp. 1115–1127.

OPTIMIZATION, SYSTEM ANALYSIS, AND OPERATIONS RESEARCH

On the Algorithm of Cargoes Transportation Scheduling in the Transport Network

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Received April 6, 2023
Revised June 19, 2023
Accepted July 20, 2023

Abstract—The problem of cargoes transportation scheduling in the transport network represented by an undirected multigraph is considered. Transportations between vertices are provided at predefined time intervals. The iterative algorithm to search for a solution approximate to the optimal one by criterion value is proposed in the problem under consideration. The algorithm is constructed on the base of solutions of mixed integer linear programming problems. The applicability of the algorithm is tested by the example with more than 90 million binary variables.

Keywords: transport network, multigraph, cargoes transportation, schedule, mixed integer linear programming

DOI: 10.25728/arcRAS.2023.74.63.001

1. INTRODUCTION

The scheduling problem (of cargoes, trains, locomotives) is a widespread problem both in theory and in practice. Publications on this topic can be divided into several groups: by the presence of movement time in the problem, by the fixedness of the movement time between vertices, by the fixedness of the movement route at optimization, by structure of the transport network (multi)graph. For example, [1] used only duration of movement time along transport network graph arcs, the graph of the special structure (one-way railway) is considered in [2, 3]. The scheduling problem for the railway network of general structure with a fixed set of routes for trains is researched in [4, 5]. The problem to construct train routes and their movement times along the railway network is solved simultaneously in [6, 7]. Time in [6, 7] is set to be discrete, that may cause to the huge dimension of the problem. The simultaneous problem of scheduling and routing for general structure railway networks is researched in [8–11]. Transportations between vertices in [8–11] are carried out at only predetermined time intervals.

Difference of problem statements with fixed movement time between vertices from problem statements with arbitrary time is very principal. In the latter there is supposed that some transport is able for the carriage at any interval of time. But it is not always physically realizable. The principal difference [11] from other researches is in possibility to not come in the arrival vertex before the end of time interval for which the timetable is scheduling (hereinafter referred as planning horizon). Such possibility is relevant when there is a cargo that needs to be departed shortly before the end of the planning horizon. But such possibility complicates not only the mathematical model of carriages but also increases the computation time [11]. That's why it is relevant to construct a faster algorithm than the algorithm from [11]. Such algorithm is being constructed in the present paper.

Within the framework of the transportation model under consideration time of readiness for departure, starting and ending times of movement of any vehicle carrying out transportation between vertices are fixed. These characteristics are real numbers. Optimization in the future will be carried out with the goal to find a particular vehicle for a particular cargo. Other optimization variables will also be considered. For example it will be the parking time of cargo at various vertices, the expected quantity of time before delivery after the end of the final planning horizon, delivery of cargo to the destination vertex.

A system from linear equalities and inequalities is formed to construct the algorithm. This system contains binary and continuous variables and sets a mathematical model for the carriage of cargoes along a transport network of general structure. The transport network is represented by an undirected multigraph. The algorithm performs a decomposition of a set of cargoes, as well as a decomposition of the planning horizon to reduce the computation time. The algorithm contains one more possibility to accelerate the computation time. This possibility is based on the cut out of transportations that are unlikely to be used by cargoes due to the beginning time of these transportations is earlier than expected arrival time in the respective to these transportations vertices. The developed algorithm is tested on a meaningful example with millions of binary variables.

2. BASIC DESIGNATIONS AND ASSUMPTIONS

Let us consider a transport network represented by an undirected multigraph $G =< V, E >$, where V is a set of vertices (cities, railway stations, plants, airports, seaports) and E is a set of edges (highways, railway tracks, seaways, airways), connecting these vertices. Let $|V| = M \geqslant 2$. By renumbering vertices of multigraph G from 1 to M, we compose a set of indices $V' = \{1, 2, \ldots, M\}$. Each element of this set uniquely determines the vertex of multigraph G. Note that the need in multigraphs for modelling transport systems follows from applications. Namely, oncoming traffic between two railway stations in the same period of time, for safety reasons, should be separated along different railway tracks. Therefore for modelling of transportations, it is necessary to separately consider all railway tracks (edges) from one vertex (station) to another (station).

We will count the time in minutes relative to a certain moment of reference. By the planning horizon we mean the time interval $[0, T_{\max})$, for which the timetable is scheduling. If the timetable is scheduled on a day (1440 minutes), then $T_{\max} = 1440$.

We divide the planning horizon into P non-overlapping intervals (half-open intervals) $\mathcal{T}_1, \ldots, \mathcal{T}_P$, i.e., $[0, T_{\max}) = \bigcup_{p=1}^{P} \mathcal{T}_p$, where $\forall p_1, p_2 \in \{1, \ldots, P\} : p_1 \neq p_2 \;\; \mathcal{T}_{p_1} \bigcap \mathcal{T}_{p_2} = \varnothing$. These intervals we will name as *partition intervals*. Let us introduce auxiliary variables $\underline{\mathcal{T}}_p \overset{\text{def}}{=} \inf \mathcal{T}_p$, $\overline{\mathcal{T}}_p \overset{\text{def}}{=} \sup \mathcal{T}_p$, $p = \overline{1, P}$. We construct sets $\mathcal{T}_1, \ldots, \mathcal{T}_P$ in such manner that

$$\underline{\mathcal{T}}_1 = 0, \;\; \overline{\mathcal{T}}_P = T_{\max}, \;\; \underline{\mathcal{T}}_{p+1} = \overline{\mathcal{T}}_p, \;\; p = \overline{1, P-1}.$$

Let us have I cargoes (parcels, containers, trains), for each of that there are given:

- index of departure vertex $v_i^{\text{dep}} \in V'$;
- index of arrival (destination) vertex $v_i^{\text{arr}} \in V'$;
- time of readiness for departure $t_i^{\text{dep}} \in [0, T_{\max})$;
- maximal amount of time d_i during which the cargo is allowed to be at the departure vertex from the moment of readiness;
- cargo travel time T_i, i.e. maximal amount of time during which the cargo is allowed to be on the transport network (excluding time at the departure vertex) computed in minutes;
- mass of the cargo $w_i \in \mathbb{R}_+$,

$i = \overline{1, I}$. The cargo is assumed to be indivisible in sense that it can not be sent in parts.

Cargoes carriages between vertices can only be carried out at certain intervals. Let K movements/transportations (by aircrafts, sea ships, trains, trucks) between vertices are available. Parameters of transportation mathematically can be represented by 7-element row $z_k \stackrel{\text{def}}{=} (v_k^{\text{beg}}, v_k^{\text{end}}, n_k, t_k^{\text{beg}}, t_k^{\text{end}}, W_k, C_k)$, where $v_k^{\text{beg}} \in V'$ is the index of starting vertex of movement, $v_k^{\text{end}} \in V'$ is the index of ending vertex of movement, moreover v_k^{beg} and v_k^{end} are indices of adjacent vertices in multigraph G, n_k is the number of the track (edge), connecting vertices with indices v_k^{beg} and v_k^{end}, $t_k^{\text{beg}} \in [0, T_{\max})$ is starting time of movement, t_k^{end} is ending time of movement, W_k is maximum transportable mass during transportation, C_k is the transportation cost of unit mass, $k = \overline{1, K}$. Let us designate using $\mathcal{Z}$ the set of all vectors z_k, $k = \overline{1, K}$. We renumber elements of set $\mathcal{Z}$ from 1 to K. Thus number from 1 to K determines the transportation and its transportation uniquely

In the future, as *timetable* of cargo we will understand the chain of transportation numbers that are used by it. One can easily determine by transportation numbers the vertices visited by the cargo, the time of visiting these vertices, edges of the multigraph used for movement, as well as other characteristics of the movement.

According to introduced partition intervals $\mathcal{T}_1, \ldots, \mathcal{T}_P$ we split the set of transportations for several parts, namely $\{1, \ldots, K\} = \mathcal{K}_1 \bigcup \mathcal{K}_2 \bigcup \ldots \bigcup \mathcal{K}_P$, where $\mathcal{K}_p \stackrel{\text{def}}{=} \{k \in \mathbb{N} : k \leqslant K, t_k^{\text{beg}} \in \mathcal{T}_p\}$, $p = \overline{1, P}$.

When transportations are carried out, the warehouses in which goods are stored can be filled. In addition some operations may be performed with cargoes, for example, repacking. Therefore we introduce minimal and maximal possible duration of stay at the vertex with index v_k^{end} after using of transportation with number k by cargo with number i: $t_{i,k}^{\text{st min}}$ and $t_{i,k}^{\text{st max}}$, $i = \overline{1, I}$, $k = \overline{1, K}$. Obviously, $\forall i = \overline{1, I}$, $k = \overline{1, K}$ $0 \leqslant t_{i,k}^{\text{st min}} \leqslant t_{i,k}^{\text{st max}}$.

Let τ_{m_1,m_2} is expected duration (starting from the moment of readiness for departure) of a cargo carriage from vertex with index m_1 to vertex with index m_2, $m_1, m_2 = \overline{1, M}$. Obviously that $\tau_{m_1,m_1} = 0$, $m_1 = \overline{1, M}$. If historical observations on carriages from vertex with index m_1 to vertex with index m_2 are available then as τ_{m_1,m_2} one can select sample mean by existing observations, $m_1, m_2 = \overline{1, M}$. If this data is unavailable then the indicated value can be estimated by an expert. Also we introduce value η_{m_1,m_2} that designates expected duration from the moment of readiness for departure to departure from vertex with index m_1 to vertex with index m_2. This value is set by analogy with τ_{m_1,m_2}, $m_1, m_2 = \overline{1, M}$.

As the route of cargo with number i we will understand the chain from transportations numbers used in series by this cargo, $i = \overline{1, I}$. As consequence one can determine the chain of vertices traversed in series by this cargo using the route. We limit the maximal quantity of transportations in the route during the planning horizon by some predetermined value J. As jth phase of the route of ith train we will mean movement of this train when there is used jth transportation in the route, $i = \overline{1, I}$, $j = \overline{1, J + 1}$. Phase $J + 1$ is technical, movement in that is not provided, it is needed for accuracy in the mathematical model formulation. We will name the vertex intermediate for ith cargo if it's neither the vertex of departure nor the vertex of arrival for that, $i = \overline{1, I}$.

We also introduce value $\mathcal{D}_i$, characterizing the denial in carriage to ith cargo: 0, when cargo is denied to carriage, 1 is otherwise, $i = \overline{1, I}$. The denial in carriage may be caused by there are not enough transportations to achieve the destination vertex with restrictions on travel time and other physical limitations. In the ideal case any of values $\mathcal{D}_i$ is equal to one, $i = \overline{1, I}$, but it is not always realizable or it was not successful to find schedule that leads to this result,

3. AUXILIARY RESULTS TO CONSTRUCT THE ALGORITHM

3.1. Mathematical Model of Movements Along Transport Network

We divide set of cargoes numbers $\mathcal{I}$ into S non-overlapping subsets $\mathcal{I}_s$, i.e. $\mathcal{I} \stackrel{\text{def}}{=} \{1, \ldots, I\} = \bigcup_{s=1}^{S} \mathcal{I}_s$, and besides $\forall s_1, s_2 \in \{1, \ldots, S\} : s_1 \neq s_2$ $\mathcal{I}_{s_1} \bigcap \mathcal{I}_{s_2} = \varnothing$. In [10–12] there was a proposal to divide set $\mathcal{I}$ by principle of having cargo numbers with the same departure and destination vertices in subsets. In addition one can only construct as many subsets as quantity of cargoes. In this case, in the subset with index 1 there will be a cargo number with the earliest/latest time of readiness for departure, with index 2—the second/penultimate time, etc.

We suppose that for every cargo with number from sets $\mathcal{I}_1, \ldots, \mathcal{I}_{\tilde{s}-1}$ there is the denial in carriage or a timetable, i.e. the chain from transportations numbers. If there is the denial in carriage for cargo with number $\hat{i} \in \bigcup_{s=1}^{\tilde{s}-1} \mathcal{I}_s$ then we assign $\hat{\delta}_{\hat{i},j,k} = 0$, $j = \overline{1, J+1}$, $k = \overline{1, K}$, and $\mathcal{D}_{\hat{i}} = 0$. If cargo with number $\hat{i} \in \bigcup_{s=1}^{\tilde{s}-1} \mathcal{I}_s$ is permitted to carriage, then value $\hat{\delta}_{\hat{i},j,k}$ is equal to one, if this cargo uses transportation with number k at the jth phase, and to zero, otherwise, $j = \overline{1, J+1}$, $k = \overline{1, K}$. At the same time we assign $\mathcal{D}_{\hat{i}} = 1$.

Initially we will construct the timetable for time interval $[0, \overline{\mathcal{T}}_1)$ to reduce the dimension of optimization problems to be solved in the future. To construct the timetable for time interval $[0, \overline{\mathcal{T}}_2) = [0, \overline{\mathcal{T}}_1) \bigcup \mathcal{T}_2$ we will take into account (freeze) the timetable for time interval $[0, \overline{\mathcal{T}}_1)$, To construct the timetable for time interval $[0, \overline{\mathcal{T}}_3) = [0, \overline{\mathcal{T}}_2) \bigcup \mathcal{T}_3$ we will take into account (freeze) the timetable for time interval $[0, \overline{\mathcal{T}}_2)$ and so on.

For this reason we consider only transportations from the beginning of the planning horizon until end of the interval $\mathcal{T}_{\tilde{p}}$, where $\tilde{p}$ is an arbitrary number from set $\{1, \ldots, P\}$. Let us formulate a set of constraints stating movements along the multigraph for cargoes with numbers from set $\mathcal{I}_{\tilde{s}}$ in this time, i.e. in the planning subhorizon $[0, \overline{\mathcal{T}}_{\tilde{p}})$. Let us suppose initially, that a timetable for cargoes with numbers from set $\mathcal{I}_{\tilde{s}}$ for subhorizon $[0, \overline{\mathcal{T}}_{\tilde{p}-1})$ $(\tilde{p} > 1)$ is not available.

By $\mathcal{K}^{\tilde{s},\tilde{p}}$ we will mean some non-empty set of transportations set $\bigcup_{p=1}^{\tilde{p}} \mathcal{K}_p$, selected for cargoes with numbers from set $\mathcal{I}_{\tilde{s}}$.

For this purpose we introduce auxiliary $\delta_{i,j,k}^{\tilde{p}}$, characterizing the usage of kth transportation by cargo with number i at jth phase when timetable is formed for the planning subhorizon $[0, \overline{\mathcal{T}}_{\tilde{p}})$, $i \in \mathcal{I}_{\tilde{s}}$, $j = \overline{1, J+1}$, $k \in \mathcal{K}^{\tilde{s},\tilde{p}}$. Variable $\delta_{i,j,k}^{\tilde{p}}$ is equal to one, if transportation with number k is used by ith cargo at jth phase, and to zero, otherwise.

We have by defnition of variables $\delta_{i,j,k}^{\tilde{p}}$

$$\delta_{i,j,k}^{\tilde{p}} \in \{0, 1\}, \quad i \in \mathcal{I}_{\tilde{s}}, \quad j = \overline{1, J+1}, \quad k \in \mathcal{K}^{\tilde{s},\tilde{p}}. \tag{1}$$

Movements of cargoes along multigraph G can be performed only along adjacent vertices

$$\sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j,k}^{\tilde{p}} v_k^{\text{end}} \leqslant \sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j+1,k}^{\tilde{p}} v_k^{\text{beg}} + \left(1 - \sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j+1,k}^{\tilde{p}}\right) M^3, i \in \mathcal{I}_{\tilde{s}}, \quad j = \overline{1, J-1}, \tag{2}$$

$$\sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j,k}^{\tilde{p}} v_k^{\text{end}} \geqslant \sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j+1,k}^{\tilde{p}} v_k^{\text{beg}} - \left(1 - \sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j+1,k}^{\tilde{p}}\right) M, i \in \mathcal{I}_{\tilde{s}}, \quad j = \overline{1, J-1}. \tag{3}$$

Let us remind that M is quantity of vertices in multigraph G. Constraints (2), (3) cause [10] to the fact that if for some $\tilde{i} \in \mathcal{I}_{\tilde{s}}$ and some $\tilde{j} \in \{1, \ldots, J\}$ it is true $\sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{\tilde{i},\tilde{j},k}^{\tilde{p}} = 0$, then $\sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{\tilde{i},j+1,k}^{\tilde{p}} = 0$, $j = \overline{\tilde{j}, J}$. If $\sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{\tilde{i},\tilde{j},k}^{\tilde{p}} = 1$, then $\sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{\tilde{i},\tilde{j}+1,k}^{\tilde{p}} = 0$ or $\sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{\tilde{i},\tilde{j}+1,k}^{\tilde{p}} = 1$. Constraints (2), (3) are identical to [10, 11] taking into account that the mathematical model is constructed for the planning subhorizon. Let us note that the third power of M in (2) ensures correctness of the mathematical model of movements along the multigraph [10].

Arrival at the destination vertex is possible in no more than J phases. Therefore we introduce constraints

$$\sum_{i \in \mathcal{I}_{\tilde{s}}} \sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,J+1,k}^{\tilde{p}} = 0. \tag{4}$$

Due to indivisibility of cargoes one can use no more than one transportation at any phase (including the first one)

$$\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{i,1,k} \leqslant 1, \quad i \in \mathcal{I}_{\tilde{s}}. \tag{5}$$

If carriage is begun then it must be performed from the respective departure vertex

$$\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{i,1,k} v^{\mathrm{beg}}_k = v^{\mathrm{dep}}_i \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{i,1,k}, \quad i \in \mathcal{I}_{\tilde{s}}. \tag{6}$$

If cargo readiness to depart happens after the upper bound of interval $\mathcal{T}_{\tilde{p}}$, then for this cargo usage of transportations are prohibited until the end of $\mathcal{T}_{\tilde{p}}$, i.e.

$$\sum_{j=1}^{J} \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{i,j,k} = 0, \quad \forall i \in \mathcal{I}_{\tilde{s}} : t^{\mathrm{dep}}_i \geqslant \overline{\mathcal{T}}_{\tilde{p}}. \tag{7}$$

Cargoes must be departed not earlier than the respective moments of readiness taking into account maximal duration of stay in departure vertices. At the same time it is possible to not depart cargo in interval $[0, \overline{\mathcal{T}}_{\tilde{p}})$, if it is admissible, taking into account maximal duration of stay in departure vertex. That's why we have constraints

$$t^{\mathrm{dep}}_i \leqslant \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{i,1,k} t^{\mathrm{beg}}_k + \left(1 - \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{i,1,k}\right) \overline{\mathcal{T}}_{\tilde{p}} \leqslant t^{\mathrm{dep}}_i + d_i, \quad \forall i \in \mathcal{I}_{\tilde{s}} : t^{\mathrm{dep}}_i < \overline{\mathcal{T}}_{\tilde{p}}. \tag{8}$$

Let us comment constraints (8). For this reason we consider cargo with number $\tilde{i} \in \mathcal{I}_{\tilde{s}} : t^{\mathrm{dep}}_{\tilde{i}} < \overline{\mathcal{T}}_{\tilde{p}}$. Due to constraints (1) and (5) there are only two possible variants: $\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{\tilde{i},1,k}$ is equal to zero or one. At the same time equality of this sum to zero (i.e. cargo with number $\tilde{i}$ is not departed) causes to the fact that the following must be true: $\overline{\mathcal{T}}_{\tilde{p}} \leqslant t^{\mathrm{dep}}_{\tilde{i}} + d_{\tilde{i}}$. If this sum is equal to one, then according to (5) only one transportation can be used and its beginning time will be in the interval $[t^{\mathrm{dep}}_{\tilde{i}}, t^{\mathrm{dep}}_{\tilde{i}} + d_{\tilde{i}}]$. It corresponds to the sense of constraints (8) introduced above.

From the same vertex cargo can only be departed once [1]

$$\sum_{j=1}^{J+1} \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}: v^{\mathrm{beg}}_k = m} \delta^{\tilde{p}}_{i,j,k} \leqslant 1, \quad i \in \mathcal{I}_{\tilde{s}}, \quad m = \overline{1, M}. \tag{9}$$

Arriving in the same vertex for cargo more than once is prohibited

$$\sum_{j=1}^{J+1} \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}: v^{\mathrm{end}}_k = m} \delta^{\tilde{p}}_{i,j,k} \leqslant 1, \quad i \in \mathcal{I}_{\tilde{s}}, \quad m = \overline{1, M}. \tag{10}$$

Departure from intermediate vertices of the route must not be earlier than arrival in these vertices. Therefore we have, taking into account minimal and maximal duration of stay, the following

$$\begin{aligned} \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{i,j,k} (t^{\mathrm{end}}_k + t^{\mathrm{st\ min}}_{i,k}) &\leqslant \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{i,j+1,k} t^{\mathrm{end}}_k \\ &+ \left(1 - \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta^{\tilde{p}}_{i,j+1,k}\right) \underline{T}, \quad i \in \mathcal{I}_{\tilde{s}}, \quad j = \overline{1, J-1}, \end{aligned} \tag{11}$$

[1] Here and below it is assumed that the sum of any variables over an empty set is equal to zero.

where

$$\underline{T} = \max_{i\in\{1,\dots,I\},k\in\{1,\dots,K\}} t_k^{\text{end}} + t_{i,k}^{\text{st min}},$$

$$\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j,k}^{\tilde{p}}(t_k^{\text{end}} + t_{i,k}^{\text{st max}}) \geqslant \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j+1,k}^{\tilde{p}} t_k^{\text{end}}, \quad i\in\mathcal{I}_{\tilde{s}}, \quad j=\overline{1,J-1}. \tag{12}$$

Constraints (11) and (12) are identical to the respective ones from [11].

To ensure allowability of parking (if it takes place) after the end of subhorizon $[0,\overline{\mathcal{T}}_{\tilde{p}})$ we impose constraints

$$\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}:v_k^{\text{end}}\neq v_i^{\text{arr}}} \delta_{i,j,k}^{\tilde{p}} \left(t_k^{\text{end}} + t_{i,k}^{\text{st max}} - \overline{\mathcal{T}}_{\tilde{p}}\right) + \overline{\mathcal{T}}_{\tilde{p}} \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j+1,k}^{\tilde{p}} \geqslant 0, \quad i\in\mathcal{I}_{\tilde{s}}, \quad j=\overline{1,J}. \tag{13}$$

To prohibit carriages after arrival in the destination vertex we use constraints

$$\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}:v_k^{\text{end}}=v_i^{\text{arr}}} \delta_{i,j,k}^{\tilde{p}} \leqslant 2\left(1-\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j+1,k}^{\tilde{p}}\right), \quad i\in\mathcal{I}_{\tilde{s}}, \quad j=\overline{1,J}. \tag{14}$$

Let us comment constraints (14). For this reason we consider cargo with number $\tilde{i}\in\mathcal{I}_{\tilde{s}}$. If this cargo arrived in the destination vertex after some phase then left part of (14) is equal to one. Therefore for compatibility of (14) it is needed that right side would be equal to zero. It means due to constraints (1) and (5) that the next after arrival phase will not be used as other phases. If cargo did not arrive in the destination vertex then left side of (14) is equal to zero. In this case the constraint is satisfied, because at any phase it is possible to use not more than one transportation. It means that right side will be equal to zero or one.

Let us introduce variable $\hat{T}_{i,j}^{\tilde{p}}$ that means duration spent by cargo with number i at jth (by order of traversing) intermediate vertex of its route during the planning subhorizon

$$\hat{T}_{i,j}^{\tilde{p}} = \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j+1,k}^{\tilde{p}}(t_k^{\text{beg}} - \overline{\mathcal{T}}_{\tilde{p}}) + \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}:v_k^{\text{end}}\neq v_i^{\text{arr}},t_k^{\text{end}}<\overline{\mathcal{T}}_{\tilde{p}}} \delta_{i,j,k}^{\tilde{p}}(\overline{\mathcal{T}}_{\tilde{p}} - t_k^{\text{end}}), \quad i\in\mathcal{I}_{\tilde{s}}, \quad j=\overline{1,J}. \tag{15}$$

We also assign $\hat{T}_{i,J+1}^{\tilde{p}}=0$ for convenience of modelling.

Further we introduce new variables $\mathcal{F}_i^{\tilde{p}}$, characterizing the expected duration of time needed until arrival in the destination vertex for cargo with number i after the end of the planning subhorizon $[0,\overline{\mathcal{T}}_{\tilde{p}})$:

$$\begin{aligned}\mathcal{F}_i^{\tilde{p}} = \tau_{v_i^{\text{dep}},v_i^{\text{arr}}} &+ \sum_{j=1}^{J}\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j,k}\left(\tau_{v_k^{\text{end}},v_i^{\text{arr}}} - \tau_{v_k^{\text{beg}},v_i^{\text{arr}}}\right)\\ &+\sum_{j=1}^{J}\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}:t_k^{\text{end}}\geqslant\overline{\mathcal{T}}_{\tilde{p}}} \delta_{i,j,k}(t_k^{\text{end}} - \overline{\mathcal{T}}_p), \quad i\in\mathcal{I}_{\tilde{s}}.\end{aligned} \tag{16}$$

Next constraints are needed to not exceed cargo travel time

$$\begin{aligned}\mathcal{F}_i^{\tilde{p}} + \sum_{j=1}^{J}\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}:t_k^{\text{end}}<\overline{\mathcal{T}}_p,v_k^{\text{end}}=v_i^{\text{arr}}} \delta_{i,j,k}^{\tilde{p}}\left(t_k^{\text{end}} - \overline{\mathcal{T}}_p\right) + \sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,1,k}^{\tilde{p}}(\overline{\mathcal{T}}_p - t_k^{\text{beg}})\\ \leqslant T_i + \left(1-\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,1,k}^{\tilde{p}}\right)\eta_{v_i^{\text{dep}},v_i^{\text{arr}}}, \quad \forall i\in\mathcal{I}_{\tilde{s}}: t_i^{\text{dep}} < \overline{\mathcal{T}}_p.\end{aligned} \tag{17}$$

Constraints (17) are identical to the respective ones from [11].

We introduce variables $\omega_i^{\tilde{p}}$, characterizing arrival of cargo with number i in the respective destination vertex on the base of used transportations during the planning subhorizon $[0, \overline{\mathcal{T}}_{\tilde{p}})$: 0—arrived, 1—did not arrive:

$$\omega_i^{\tilde{p}} = 1 - \sum_{j=1}^{J} \sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}: t_k^{\text{end}} < \overline{\mathcal{T}}_p, v_k^{\text{end}} = v_i^{\text{arr}}} \delta_{i,j,k}^{\tilde{p}}, \quad i \in \mathcal{I}_{\tilde{s}}. \tag{18}$$

Next constraints are caused by the need in not exceeding maximal allowable mass at transportation with number k

$$\sum_{i \in \mathcal{I}_{\tilde{s}}} \sum_{j=1}^{J+1} \delta_{i,j,k}^{\tilde{p}} w_i \leqslant W_k - \sum_{i \in \bigcup\limits_{s=1}^{\tilde{s}-1} \mathcal{I}_s} \sum_{j=1}^{J+1} \hat{\delta}_{i,j,k} w_i, k \in \mathcal{K}^{\tilde{s},\tilde{p}}. \tag{19}$$

3.2. Optimality Criterion

Potentially system of equalities and inequalities (1)–(19) may not have a unique solution. Therefore a criterion is required to select among solutions. Let us compose from all $\delta_{i,j,k}^{\tilde{p}}$ vector $\delta^{\tilde{s},\tilde{p}}$, $i \in \mathcal{I}_{\tilde{s}}$, $j = \overline{1, J+1}$, $k \in \mathcal{K}^{\tilde{s},\tilde{p}}$. Also we compose from all $\mathcal{F}_i^{\tilde{p}}$ vector $\mathcal{F}^{\tilde{s},\tilde{p}}$, from $\omega_i^{\tilde{p}}$ vector $\omega^{\tilde{s},\tilde{p}}$, $i \in \mathcal{I}_{\tilde{s}}$. We unite all $\hat{T}_{i,j}^{\tilde{p}}$ in vector $\hat{T}^{\tilde{s},\tilde{p}}$, $i \in \mathcal{I}_{\tilde{s}}$, $j = \overline{1, J+1}$.

Let us choose the criterial function of the following form

$$\begin{aligned} & J_{\tilde{s}}^{\tilde{p}} \left(\delta^{\tilde{s},\tilde{p}}, \mathcal{F}^{\tilde{s},\tilde{p}}, \omega^{\tilde{s},\tilde{p}}, \hat{T}^{\tilde{s},\tilde{p}} \right) \\ &= c_1 \underbrace{\sum_{i \in \mathcal{I}_s} \sum_{j=1}^{J+1} \sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j,k}^{\tilde{p}} (\min\{t_k^{\text{end}}, \overline{\mathcal{T}}_{\tilde{p}}\} - t_k^{\text{beg}})}_{\substack{\text{the total time in movement} \\ \text{during the planning subhorizon}[0,\overline{\mathcal{T}}_{\tilde{p}})}} + c_2 \underbrace{\sum_{i \in \mathcal{I}_s} \sum_{j=1}^{J+1} \hat{T}_{i,j}^{\tilde{p}}}_{\substack{\text{the total parking} \\ \text{time in} \\ \text{intermediate} \\ \text{vertices}}} \\ &+ c_3 \underbrace{\sum_{i \in \mathcal{I}_s} \left(\sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,1,k}^{\tilde{p}} t_k^{\text{beg}} + \left(1 - \sum_{k=1}^{K} \delta_{i,1,k}^{\tilde{p}} \right) \overline{\mathcal{T}}_{\tilde{p}} - t_i^{\text{dep}} \right)}_{\substack{\text{the total parking time in departure vertices} \\ \text{from the time of readiness for departure} \\ \text{until the end of the planning subhorizon}[0,\overline{\mathcal{T}}_{\tilde{p}})}} \\ &+ c_4 \underbrace{\sum_{i \in \mathcal{I}_s} \sum_{j=1}^{J+1} \sum_{k \in \mathcal{K}^{\tilde{s},\tilde{p}}} \delta_{i,j,k}^{\tilde{p}} w_i C_k}_{\substack{\text{the total cost} \\ \text{of transportations}}} + c_5 \underbrace{\sum_{i \in \mathcal{I}_s} \mathcal{F}_i^{\tilde{p}}}_{\substack{\text{the total} \\ \text{expected} \\ \text{time until} \\ \text{delivery}}} + c_6 \underbrace{\sum_{i \in \mathcal{I}_s} \omega_i^{\tilde{p}}}_{\substack{\text{the total} \\ \text{quantity} \\ \text{of undelivered} \\ \text{cargoes during} \\ \text{the planning} \\ \text{subhorizon}}}, \end{aligned} \tag{20}$$

where $c_1, \ldots, c_6$ are non-negative values chosen by a decision-maker. The choice of $c_1, \ldots, c_6$ impacts on the sense of optimization. When $c_1 = c_2 = c_3 = c_5 = c_6 = 0$, $c_4 = 1$ there is a problem to minimize the total cost of transportations. When $c_1 = c_2 = c_3 = c_5 = 1$, $c_4 = c_6 = 0$ there is a problem to minimize the sum of already spent time by cargoes in the transport network within the planning subhorizon and the expected time to delivery after the end of the planning subhorizon. We will mean as rth *criterion component* multiplier of c_r in (20), $r = \overline{1, 6}$. It should be noted that not all criterion components are homogeneous. The first, second, third and fifth are measured in minutes, the fourth is in units of cost, and the sixth is in pieces. If optimization problem is connected only with homogeneous components, then dimension of coefficients $c_1, \ldots, c_6$ is not important. If it is needed for optimization to take into account heterogeneous components then problem to minimize the total cost should be considered, i.e. values c_1, c_2, c_3, c_5 will be measured in units of cost/minute, and c_6 in units of cost/piece.

If one takes $\tilde{p} = P$, then the planning subhorizon will coincide with $[0, \overline{\mathcal{T}}_{\tilde{p}})$. If we do not split cargoes numbers set, i.e. $\mathcal{I} = \mathcal{I}_{\tilde{s}}$ and $\mathcal{K}^{\tilde{s},P} = \bigcup_{p=1}^{P} \mathcal{K}_p$, then criterion (20) and system of constraints (1)–(19) will be precisely the same as criterion and system of constraints in [11]. But for this split (more accurately—for the absence of the split) of cargoes numbers set and value of $\tilde{p}$, that is suitable to decrease quantity elements in transportation set, used for scheduling, direct optimization of criterion (20) with the purpose to find a timetable on the entire planning horizon may be very prolonged. That's why we will form the algorithm to search although not optimal but relatively fast solution on the base of obtained in the paper constraints.

The presence of linear on optimization variables constraints (2)–(19) and linear criterion (20), binary variables vectors $\delta^{\tilde{s},\tilde{p}}$ and $\omega^{\tilde{s},\tilde{p}}$, real variable vectors $\mathcal{F}^{\tilde{s},\tilde{p}}$ and $\hat{T}^{\tilde{s},\tilde{p}}$ makes problem (20) with constraints (1)–(19) mixed integer linear programming problem.

4. THE ALGORITHM FOR SCHEDULING

At formation of the algorithm we will take into account the possibility of more fast computation time by cut out of transportations that are unlikely to be used by cargoes.

It makes no sense at scheduling for a given subhorizon to take into account transportations from vertices to which none of the cargo in this subhorizon will arrive. Generally speaking, in order to determine whether a particular cargo will vertex a specific vertex in a given time, it is necessary to solve the corresponding optimization problem. However, solving these types of problems takes time. Therefore, to establish the fact that the loads will not arrive a certain vertex, we will use the values τ_{m_1,m_2}, $m_1 = \overline{1, M}$, $m_2 = \overline{1, M}$. Of course, the conclusion on possibility to arrive at a certain vertex based on the values τ_{m_1,m_2} is not always true, $m_1 = \overline{1, M}$, $m_2 = \overline{1, M}$. This is because these values are based on past transportation history rather than the transportation currently available. Nevertheless, this significantly reduces the computation time, although with a deterioration in the value of the criterion function/inability to accept some cargoes for transportation. We will compare values τ_{m_1,m_2} with the ratio of the length of the corresponding partition interval to an acceleration parameter, $m_1 = \overline{1, M}$, $m_2 = \overline{1, M}$. The acceleration parameter, which is dimensionless, will be denoted by A. The lower A the less transportations will be crossed out, but the more cargoes will likely be accepted for carriage. And, on the contrary, the larger A the faster computation time will be, but the quality (in terms of cargoes accepted for carriage) of obtained solution will be worse. If $A = 0$ there will be no strikeouts. When solving optimization problems, it seems most rational to set A equal to one. In this case, the expected time before arrival at a certain vertex will be compared with the duration of the corresponding partition interval, i.e. a period of time in which the timetable has not yet been frozen and is being searched.

1. Values $c_1, \ldots, c_6 \in \mathbb{R}_+$ are initialized. Numbers P, $J \in \mathbb{N}$ are stated. The number $A \in \mathbb{R}_+$ is set.

2. Set of cargoes numbers is divided into $S \in \mathbb{N}$ non-overlapping subsets $\mathcal{I}_s$, i.e. $\{1, \ldots, I\} = \bigcup_{s=1}^{S} \mathcal{I}_s$, and besides $\forall s_1, s_2 \in \{1, \ldots, S\} : s_1 \neq s_2$ $\mathcal{I}_{s_1} \bigcap \mathcal{I}_{s_2} = \varnothing$.

3. Partition intervals $\mathcal{T}_1, \ldots, \mathcal{T}_P$ are formed in such manner, that $[0, T_{\max}) = \bigcup_{p=1}^{P} \mathcal{T}_p$, where $\forall p_1, p_2 \in \{1, \ldots, P\} : p_1 \neq p_2$ $\mathcal{T}_{p_1} \bigcap \mathcal{T}_{p_2} = \varnothing$, and besides $\underline{\mathcal{T}}_1 = 0$, $\overline{\mathcal{T}}_P = T_{\max}$, $\underline{\mathcal{T}}_{p+1} = \overline{\mathcal{T}}_p$, $p = \overline{1, P-1}$.

4. Sets $\mathcal{K}_p = \{k \in \mathbb{N} : k \leqslant K, t_k^{\text{beg}} \in \mathcal{T}_p\}$ are formed, $p = \overline{1, P}$.

5. Parameter $\tilde{s}=1$ is initialized by 1.

6. Parameter $\tilde{p}=1$ is initialized by 1.

7. If $\tilde{p}$ is equal to one, then set $\mathcal{V}^{\tilde{s},\tilde{p}}=\bigcup_{i\in\mathcal{I}_{\tilde{s}}} v_i^{\mathrm{dep}}$ is formed. If $\tilde{p}$ is greater than one, then

$$\mathcal{V}^{\tilde{s},\tilde{p}}=\bigcup_{i\in\mathcal{I}_{\tilde{s}}}\begin{cases} v_i^{\mathrm{dep}}, & \sum\limits_{j=1}^{J+1}\sum\limits_{k\in\mathcal{K}^{\tilde{s},\tilde{p}-1}}\overline{\delta}_{i,j,k}^{\tilde{p}-1}=0,\\ \sum\limits_{k\in\mathcal{K}^{\tilde{s},\tilde{p}-1}}\overline{\delta}_{i,j_i,k}^{\tilde{p}-1}v_k^{\mathrm{end}}, & \sum\limits_{j=1}^{J+1}\sum\limits_{k\in\mathcal{K}^{\tilde{s},\tilde{p}-1}}\overline{\delta}_{i,j,k}^{\tilde{p}-1}>0,\end{cases}$$

where

$$j_i=\sum_{j=1}^{J+1}\sum_{k\in\mathcal{K}^{\tilde{s},\tilde{p}-1}}\overline{\delta}_{i,j,k}^{\tilde{p}-1},\quad i\in\mathcal{I}_{\tilde{s}}.$$

Set $\mathcal{V}^{\tilde{s},\tilde{p}}$ consists of departure vertices indices for those cargoes that have not been in movement yet and from indices of last (on the current moment) vertices for those cargoes that had at least one transportation.

8. If $A=0$, then $\mathcal{K}^{\tilde{s},\tilde{p}}=\mathcal{K}_{\tilde{p}}$. If $A>0$, then set $\mathcal{K}^{\tilde{s},\tilde{p}}=\Big\{k\in\mathcal{K}_{\tilde{p}}:\min\limits_{m\in\mathcal{V}^{\tilde{s},\tilde{p}}}\tau_{m,v_k^{\mathrm{beg}}}\leqslant(\overline{\mathcal{T}}_{\tilde{p}}-\underline{\mathcal{T}}_{\tilde{p}})/A,$ $\min\limits_{i\in\mathcal{I}_{\tilde{s}}}t_i^{\mathrm{dep}}\leqslant t_k^{\mathrm{beg}}\Big\}$ is formed.

9. If $\tilde{p}>1$, then set $\mathcal{K}^{\tilde{s},\tilde{p}}=\bigcup_{p=1}^{\tilde{p}-1}\mathcal{K}^{\tilde{s},p}\bigcup\mathcal{K}^{\tilde{s},\tilde{p}}$ is formed. If $\tilde{p}=1$, then $\mathcal{K}^{\tilde{s},\tilde{p}}=\mathcal{K}^{\tilde{s},\tilde{p}}$.

10. If set $\mathcal{K}^{\tilde{s},\tilde{p}}$ is empty and $\tilde{p}<P$, then value $\tilde{p}$ is increased by 1, go to step 7.

If set $\mathcal{K}^{\tilde{s},\tilde{p}}$ is empty and $\tilde{p}=P$, then $\hat{\delta}_{i,j,k}=0$, $\mathcal{D}_i=0$, $i\in\mathcal{I}_{\tilde{s}}$, $j=\overline{1,J+1}$, $k=\overline{1,K}$. If $\tilde{s}=S$, then the algorithm is finished. If $\tilde{s}<S$, then value $\tilde{s}$ is increased by 1, go to step 6.

If set $\mathcal{K}^{\tilde{s},\tilde{p}}$ is not empty, go to step 11.

11. The problem

$$J_{\tilde{s}}^{\tilde{p}}(\delta^{\tilde{s},\tilde{p}},\mathcal{F}^{\tilde{s},\tilde{p}},\omega^{\tilde{s},\tilde{p}},\hat{T}^{\tilde{s},\tilde{p}})\to\min_{\delta^{\tilde{s},\tilde{p}},\mathcal{F}^{\tilde{s},\tilde{p}},\omega^{\tilde{s},\tilde{p}},\hat{T}^{\tilde{s},\tilde{p}}}$$

with constraints (1)–(19), and also (when $\tilde{p}>1$) constraint

$$\delta_{i,j,k}^{\tilde{p}}=\overline{\delta}_{i,j,k}^{\tilde{p}-1},\quad i\in\mathcal{I}_{\tilde{s}},\quad j=\overline{1,J+1},\quad k\in\mathcal{K}^{\tilde{s},\tilde{p}-1}\tag{21}$$

is solved.

If a solution of this problem does not exist then $\hat{\delta}_{i,j,k}=0$, $\mathcal{D}_i=0$, $i\in\mathcal{I}_{\tilde{s}}$, $j=\overline{1,J+1}$, $k=\overline{1,K}$. If $\tilde{s}=S$, then the algorithm is finished. If $\tilde{s}<S$, then value $\tilde{s}$ is increased by 1, go to step 6.

If a solution was found and $\tilde{p}<P$, then values $\overline{\delta}_{i,j,k}^{\tilde{p}}$ are set: $\overline{\delta}_{i,j,k}^{\tilde{p}}$ is equal to one, if cargo with number i at jth phase uses transportation with number k, and is equal to zero, otherwise, $i\in\mathcal{I}_{\tilde{s}}$, $j=\overline{1,J+1}$, $k\in\mathcal{K}^{\tilde{s},\tilde{p}}$. Value $\tilde{p}$ is increased by 1, go to step 7.

If a solution was found and $\tilde{p}=P$, then $\mathcal{D}_i=1$, values $\hat{\delta}_{i,j,k}$ are set: $\hat{\delta}_{i,j,k}$ is equal to one, if cargo with number i at jth phase uses transportation with number k, and is equal to zero, otherwise, $i\in\mathcal{I}_{\tilde{s}}$, $j=\overline{1,J+1}$, $k=\overline{1,K}$. If $\tilde{s}=S$, then the algorithm is finished. If $\tilde{s}<S$, then value $\tilde{s}$ is increased by 1, go to step 6.

Is should be noted that constraint (21) allows to freeze the timetable for time interval $[0,\overline{\mathcal{T}}_1)$ when the timetable for interval $[0,\overline{\mathcal{T}}_2)$ is searched, the timetable for time interval $[0,\overline{\mathcal{T}}_2)$ when the timetable for interval $[0,\overline{\mathcal{T}}_3)$ is searched and so on.

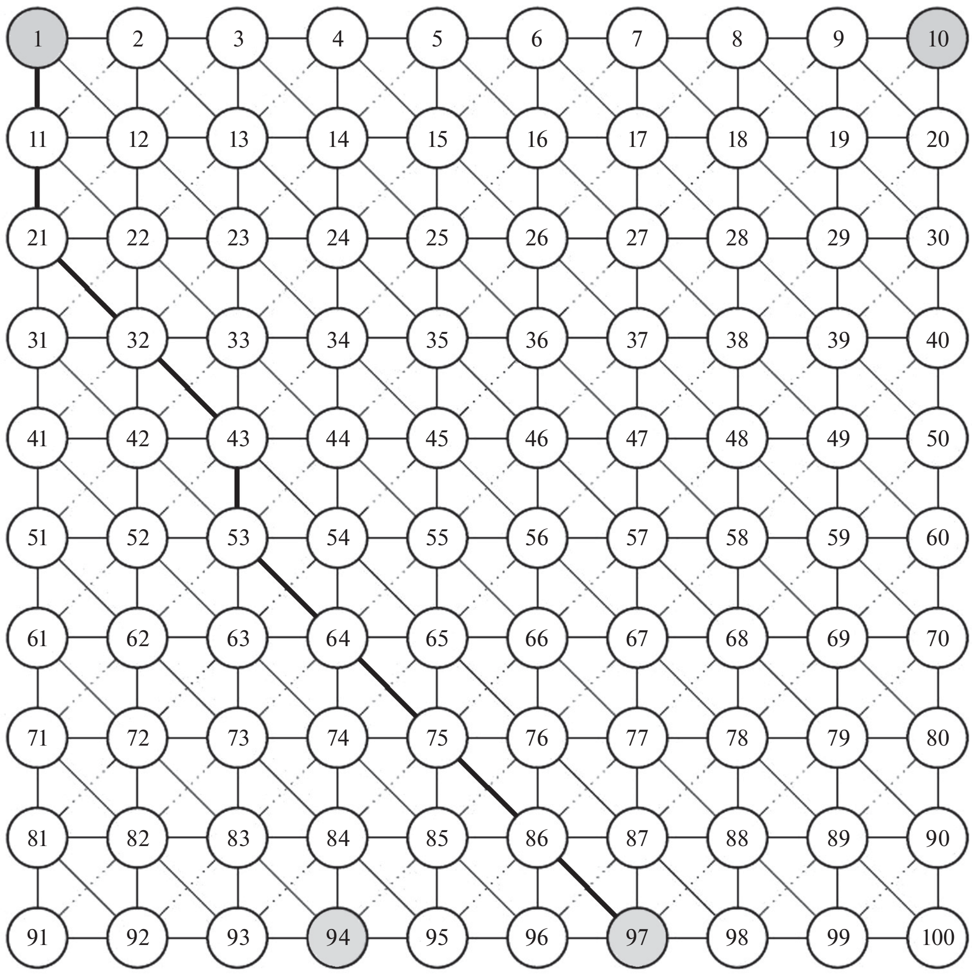
As the *minimal/maximal time* algorithm we will name such version of the proposed above algorithm when at step 2 the split is carried out in ascending and descending order of cargoes readiness moments for departure. Namely, set $\mathcal{I}_1$ consists of cargo number with the earliest/latest time of readiness for departure, set $\mathcal{I}_2$—with the second/penultimate and so on.

5. THE EXAMPLE

Let us consider a model example.

Let the multigraph of the transport network has the form shown in figure. For greater clarity the second track (the second edge) between adjacent vertices is omitted. The graph shows tracks with number 1. Some edges are indicated by a dashed line to show the multilevel intersection of edges in the transport network.

Suppose that some point of reference is chosen and $T_{\max} = 1440$ minutes. Starting from the point of reference: 5 cargoes of the same mass in 1 unit appear every 60 minutes at the vertex with index 1, these cargoes need to be transported to the vertex with index 97; 5 cargoes of the same mass in 1 unit appear every 60 minutes at the vertex with index 10, these cargoes need to be transported to the vertex with index 94.



Multigraph G of the transport network (by orange color departure and destination vertices are highlighted, by blue color the most frequent path of delivered cargoes for the one of the obtained results are highlighted).

Table 1. Properties of an approximate solution found by minimal time algorithm in the format: the total carriages time/the quantity of cargoes accepted for delivery/the quantity of delivered cargoes/the total cost of carriages/the computation time in minutes for various P and J

$P \backslash J$	9	12	15
6	147 960/192/68/21 550/75	**183 855**/240/122/28 840/91	**183 600**/240/122/28 840/99
12	141 675/182/60/19 720/45	**186 435**/240/117/28 520/52	**186 435**/240/117/28 520/58
24	170 230/220/94/25 200/54	**195 590**/240/96/28 160/59	**195 590**/240/96/28 160/64

Table 2. Properties of an approximate solution found by minimal time algorithm in the format: the total carriages time/the quantity of cargoes accepted for delivery/the quantity of delivered cargoes/the total cost of carriages/the computation time in minutes for various P and J

$P \backslash J$	9	12	15
6	148 475/192/64/21 530/73	**183 580**/240/122/29 040/88	**183 580**/240/122/29 040/107
12	124 245/156/26/15 560/50	187 870/236/106/28 060/54	**191 685**/240/110/28 770/62
24	171 470/222/96/25 760/55	171 470/222/96/25 760/57	171 470/222/96/25 760/62

Tranportations between vertices with an index difference equal to 1 or 10 by the absolute value are carried out every 30 minutes, cost of such transportations is 10 per unit of mass, maximal mass to transport is 2 units, duration of transportation is 60 minutes. Tranportations between vertices with an index difference equal to 9 or 11 by the absolute value are carried out every 30 minutes, cost of such transportations is 20 per unit of mass, maximal mass to transport is 2 units, duration of transportation is 85 minutes. Thus $I = 240$, $K = 32\,832$, $M = 100$.

Suppose also that $d_i = 180$, $T_i = 960$, $t_{i,k}^{\text{st min}} = 0$, $t_{i,k}^{\text{st max}} = 120$, $i = \overline{1, I}$, $k = \overline{1, K}$.

Suppose $\eta_{m_1,m_2} = 0$, $m_1, m_2 = \overline{1, 100}$. Let

$$\tau_{m_1+1,m_2+1} = \begin{cases} 90, \quad |m_1\%10 - m_2\%10| = 1 \text{ and } |\lfloor m_1/10\rfloor - \lfloor m_2/10\rfloor| = 1 \\ 60|m_1\%10 - m_2\%10| + 60|\lfloor m_1/10\rfloor - \lfloor m_2/10\rfloor|, \quad \text{otherwise,} \end{cases}$$

where $x\%y$ is remainder of x divided by y, $\lfloor x\rfloor$ is the integer part of x, $m_1, m_2 = \overline{0, 99}$. Such choice of values τ_{m_1,m_2} provides that expected carriage duration from one adjacent vertex to another (if they are connected diagonally) is 90 minutes. In all other cases the expected carriage duration is proportional to the minimum number of edges when travelling from one vertex to another is carried out without using diagonal edges.

We consider the case where $c_1 = c_2 = c_3 = c_5 = 1$, $c_4 = c_6 = 0$. We set $A = 1$. Let us analyze, how results of applying proposed algorithms depend on P and J. The duration of intervals $\mathcal{T}_1, \ldots, \mathcal{T}_P$ will be the same. Let us preliminarily note that with available transportations, direct carriage from the vertex with index 1 to the vertex with index 12 costs the same as carriage through the intermediate vertex with index 2 (or index 11), while carriage directly takes less time. However, the fastest carriage from departure vertices–diagonally–due to the declared maximal mass and the frequency of transportations is not available for every cargo, so the optimization problem, generally speaking, is non-trivial.

In Tables 1 and 2 by bold font there are highlighted cases where all cargoes were accepted for delivery. As follows from Tables 1 and 2 the best result was obtained for maximal time algorithm with $P = 6$, $J = 12$. This solution we will name *basic*. For the basic solution the most frequent chain of vertices indices traversed at movement by delivered cargoes is

$$1 \to 11 \to 21 \to 32 \to 43 \to 53 \to 64 \to 75 \to 86 \to 97.$$

Table 3. Further improve of obtained solution by maximal time algorithm

Parameters of the algorithm	The total time of carriages	The quantity of cargoes accepted for delivery	The quantity of delivered cargoes	The total cost of carriages	The computation time, minutes
$A = 1, P = 4, J = 12$	**182 455**	240	122	28 830	222
$A = 0{,}5, P = 6, J = 12$	**183 165**	240	124	29 000	187

This chain appeared for 8 cargoes. Among delivered during the planning horizon cargoes: for 74 cargoes there were used 9 transportations, for 41 cargoes there were used 10 transportations, for 7 cargoes there were used 11 transportations. Exactly half of delivered cargoes was sent from the vertex with index 1.

Another result of the study is the fact that for cases when all cargoes are accepted for delivery, with a fixed J with decreasing P the computation time (as expected) increases, since mathematical programming problems of higher dimension are solved. Decreasing in the criterion is also observed. The growth of J at a fixed P causes to the fact that more cargoes are accepted for delivery. However, an increase in J from 12 to 15 in this problem did not allow us to decrease the criterion value always. This observation can be caused by the fact that T_i is relatively small, $i = \overline{1, I}$. Therefore, routes with a large number of transportations and travel time from the moment of readiness can not be used. In addition, the goal of optimization is to minimize the total travel time, and diagonal movement, as noted earlier, faster.

Note that even at $J = 12$ taking into account constraint (4) there are $I \cdot J \cdot K = 94\,556\,160$ binary variables in the problem under study. At the same time the solution search time is about an hour, which can be considered as an acceptable speed. To speed up the search for a solution, one can, for example, fix a certain set of vertices through which this or that cargo must travel. If a timetable is found for this set in such manner, then it is possible not to search for a timetable for this set of cargoes on the entire set of transportations. It is also possible to reduce the number of elements in the set $\mathcal{K}^{\tilde{s},\tilde{p}}$, formed at the 8th step of the proposed algorithm, $\tilde{s} = \overline{1, S}$, $\tilde{p} = \overline{1, P}$. For example, one can exclude transportations with starting or ending vertices that have already been visited by all cargoes from the set $\mathcal{I}_{\tilde{s}}$, $\tilde{s} = \overline{1, S}$. However, such (and similar) modifications, leading to the increasing of the obtaining solution speed, may degrade the solution in terms of quality.

Let us investigate the question about quality of basic solution. For this purpose we reduce P or A.

As follows from Table 3 decreasing A and P allowed to find a bit better (around 0.5 %) solution by criterion value than basic solution. But the search time for any of improved solutions has increased several times. Increasing computation time was caused by increasing dimension of solved problems at the algorithm work.

Note that the proposed algorithm can potentially be used not only for strategic but also operational planning. Operational planning is possible in situations with fewer transports/fewer multigraph vertices than those considered in this example [11]. The question of the maximum dimension of the problem being solved, at which operational planning is possible using the developed algorithm, is of separate scientific interest. It must be said that it is possible to speed up the work of the proposed algorithm with a new/different version of the mixed integer linear programming problem solver.

All results were obtained using ILOG CPLEX 12.5.1 mathematical package on the personal computer (Intel Core i5 4690, 3.5 GHz, 8 GB DDR3 RAM).

6. CONCLUSION

In this paper we have studied the problem of cargoes transportation scheduling in the transport network represented by the undirected multigraph. Transportation between vertices were carried out at predetermined time intervals. To solve this problem the mathematical model of carriages along a multigraph was proposed. This model was constructed using linear equalities and inequalities containing binary and continuous variables. The optimization criterion was formulated. The algorithm to find an approximate solution was proposed due to possible high dimension of the obtained problem. The algorithm is based on the decomposition of the cargoes set and the planning horizon. Additionally, a parameter is introduced into the algorithm for the acceleration of its work. This parameter controls the number of transportations on which the timetable is built at one or another step of the algorithm. A study of the quality of decomposition was carried out on a meaningful example with millions of binary variables.

FUNDING

This work was supported by the Russian Science Foundation, project no. 23-21-00293.

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This paper was recommended for publication by B.M. Miller, a member of the Editorial Board